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Conformally Exact Black Hole Perturbed by a Marginal Operator

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abstract

We have examined effective theory induced by gauged WZW models, in which the tachyon field is added as a marginal operator. Due to this operator added, we must further add the higher order corrections, which modifies the original configuration, to make the theory full-conformally invariant. It has been found that 2d is a critical dimension in the sense that the metric obtained from gauged WZW is modified by the tachyon condensation for $d > 2$, but not for $d \leq 2$.

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1 Introduction

Conformal field theories of cosets based on a gauged WZW model [1] have provided several new solutions of string equations. Some solutions have been also found by exploiting duality transformations [2] of the known solutions. These approaches opened a new direction for learning more about particles and string theories in the gravitationally singular configurations, e.g., black hole, black string and black branes. However they give semi-classical string solutions, which are available for $k \rightarrow \infty$, where k represents the level of a WZW model.

However the conformal invariance of the theory is realized at a finite k , so we need a solution which is exact to all orders of $1/k$ expansion. A method to get such exact metric and dilaton has been proposed in [3], [4], and the analyses according to this method have been given in [5],[6]. The idea is to identify the Virasoro operator $L_0 + \bar{L}_0$ with the Klein-Gordon operator for the tachyon. Although this method is intuitive, the exactness of the solutions was checked perturbatively to three [7] and four loops [8] for the $SL(2, R)/U(1)$ case. It has also been suggested [9][10] how to get the exact solutions from the functional integral approach.

All the solutions obtained in these ways are however given for the zero tachyon field. While the tachyon is expected to be condensed to form a background configuration from the analysis of the matrix model and other approach [11]. So it is important to examine the singular background with non-trivial tachyon configuration in order to get a more realistic insight of particle and string theories in a singular configurations. The purpose of this note is to study the possibility of a space-time background with singularity like a black hole with a condensed tachyon field according to a perturbative method [12] by starting from a solution of a gauged WZW. In other words, we could see the back reaction of the tachyon condensation to the singular solutions given by gauged WZW models.

2 Gauged WZW and Perturbation

A method of deriving the exact string solutions from gauged WZW models directly at the $2d$ field theory level has been proposed [9]. Start with the following WZW action

$$S_W(g) = \frac{1}{4\pi} \int_{\Sigma} Tr(g^{-1} \partial_+ g g^{-1} \partial_- g) - \frac{1}{12\pi} \int_B Tr(g^{-1} dg \wedge g^{-1} dg \wedge g^{-1} dg), \quad (1)$$

where $g \in G$ and $\partial B = \Sigma$. By gauging a subgroup H of G , we arrive at the action which is written in a combination of two WZW action. Then the fully corrected action can be obtained within a WZW. Then the final result is obtained in the form

of a non-linear sigma model,

$$S = \frac{1}{4\pi} \int d^2z \sqrt{g} [(G_{\mu\nu}^{(0)}(X) + B_{\mu\nu}^{(0)}(X))(g^{\alpha\beta} + i\epsilon^{\alpha\beta})\partial_\alpha X^\mu \partial_\beta X^\nu + R\Phi^{(0)}(X)] \quad , \quad (2)$$

where X^μ are the invariant elements of g .² However this story seems to be incomplete since unwanted nonlocal terms appear in the final stage, eq.(2).

Then it is convenient to proceed in an alternative way [3], where the linear $L_0 + \bar{L}_0$ constraint on the tachyon field $T(X)$ is identified with the Klein-Gordon operator,

$$(L_0 + \bar{L}_0)T = -\frac{1}{e^{-2\Phi}\sqrt{G}}\partial_\mu G^{\mu\nu} e^{-2\Phi}\sqrt{G}\partial_\nu T, \quad (3)$$

where the Virasoro operators L_0 and \bar{L}_0 are defined for the coset G/H . Since they are expressed by the differential operators on the group parameter space, we can read off the exact metric and the dilaton from eq.(3). However the antisymmetric tensor can not be obtained from this method. But we need not it in the analysis hereafter.

We notice that eq.(2) can be interpreted as a fully corrected effective action of a 2d gravity which couples to some renormalizable matter system. In the case of 2d gravity with conformal matters [13] [14], we know the following solutions,

$$\{G_{\mu\nu}^{(0)}, B_{\mu\nu}^{(0)}, \Phi^{(0)}\} = \{\delta_{\mu\nu}, 0, \frac{1}{2}Q_c X^0\}, \quad (4)$$

where X^0 is the conformal mode, and the number Q_c is determined by the central charge of the theory. This configuration is called as the linear dilaton vacuum. A perturbative way to search for a more complicated background is proposed [12] by taking an action, which is made of (4) and the tachyon part as a marginal operator, as the starting point. This added tachyon part is corresponding to the interacting term of the matter fields on the world surface. We extend this method to the case, where the configuration of (4) is replaced by a solution given by a gauged WZW, in order to investigate the singular configurations which may coexist with a background tachyon.

In general, the string solutions are obtained by solving the equations of zero β -functions of the following action with the tachyon term $T(X)$,

$$S_{eff} = \frac{1}{4\pi} \int d^2z \sqrt{g} \left[\frac{1}{2} G_{\mu\nu}(X) g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu + R\Phi(X) + T(X) \right], \quad (5)$$

where $G_{\mu\nu}$ and Φ are different from $G_{\mu\nu}^{(0)}$ and $\Phi^{(0)}$ due to the non-trivial T . The antisymmetric tensor $B_{\mu\nu}$ has been dropped since it is not essential here.

² The parametrization of the group elements and which is the invariant element are shown below.

Our strategy is as follows; (a):Firstly consider an exact solution , $G_{\mu\nu}^{(0)}$ and $\Phi^{(0)}$, which are obtained from a gauged WZW model for $T = 0$. (b):Then solve the linearized equation of motion for T under the background given in (a). (c):Then solve the equations for $G_{\mu\nu}$ and Φ by using the solution of T given in the step (b), and estimate their deviations from the original solutions, $G_{\mu\nu}^{(0)}$ and $\Phi^{(0)}$.

However there is a technical problem in this program. Since we start from a non-trivial metric $G_{\mu\nu}^{(0)}$, which provides the singularity of the curvature, we need the exact form of equations of $G_{\mu\nu}$ and Φ to perform the step (c). But it is impossible to write down them. So we concentrate our attention on the region far from the singularity in order to perform the step (c) in terms of an approximate target space action given below. The distance from the singularity is characterized by a mass scale (μ) which is related to the black hole mass. In this region we can investigate the equations by expanding $G_{\mu\nu}^{(0)}$ and $\Phi^{(0)}$ around the linear dilaton vacuum (4).³ Therefore the following approximate target space action [12], [11], [16] is useful in this region,

$$S_t = \frac{1}{4\pi} \int d^d X \sqrt{G} e^{-2\Phi} [R - 4(\nabla\Phi)^2 + \frac{1}{16}(\nabla T)^2 + \frac{1}{16}v(T) - \kappa], \quad (6)$$

where the higher derivative terms are suppressed, and

$$v(T) = -2T^2 + \frac{1}{6}T^3 + \dots \quad (7)$$

∇_μ denotes the covariant derivative with respect to the metric $G_{\mu\nu}^{(0)}$. From S_t , we obtain the following equations,

$$\nabla^2 T - 2\nabla\Phi\nabla T = \frac{1}{2}v'(T), \quad (8)$$

$$\nabla^2\Phi - 2(\nabla\Phi)^2 = -\frac{\kappa}{2} + \frac{1}{32}v(T), \quad (9)$$

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R = -2\nabla_\mu\nabla_\nu\Phi + G_{\mu\nu}\nabla^2\Phi + \frac{1}{16}\nabla_\mu T\nabla_\nu T - \frac{1}{32}G_{\mu\nu}(\nabla T)^2. \quad (10)$$

We must notice that $G_{\mu\nu}^{(0)}$ and $\Phi^{(0)}$ do not satisfy eqs.(9) and (10) even if we take $T = 0$ since these equations are the approximate one. However they are correct up to the order of $O(\mu^2)$ when we expand them in power series of μ^2 , so we can see the modifications of $G_{\mu\nu}^{(0)}$ and $\Phi^{(0)}$ due to $T^{(0)}$ by solving these equations up to the same order.

³We can see below that expanding the solution in terms of μ by assuming its smallness is equivalent to the expansion of the configuration far away from the singularity.

3 **d=2; $SL(2, R)/U(1)$ model**

According to the axial gauge invariant formalism, we can arrive at the well-known 2d black hole configuration. Here we take the following parametrization,

$$g = \exp(\frac{i}{2}\theta_L\sigma_2) \exp(\frac{1}{2}r\sigma_1) \exp(\frac{i}{2}\theta_R\sigma_2), \quad (11)$$

and $X^\mu = (r, \theta)$, where $\theta \equiv \frac{1}{2}(\theta_L - \theta_R)$. Then we can get the following configurations,

$$G_{\mu\nu}^{(0)} = \frac{k-2}{2} \text{diag}(1, \beta(r)), \quad \beta(r) = \frac{4}{\coth^2(\frac{r}{2}) - \frac{2}{k}} \quad (12)$$

$$-2\Phi^{(0)} = a + \frac{1}{2} \ln [\sinh^2(r)/\beta(r)] = \text{const} + \ln(\sinh r) - \frac{1}{2} \ln G^{(0)}, \quad (13)$$

where $G^{(0)} = \det G_{\mu\nu}^{(0)}$ and a is a constant.

It is instructive to expand this configuration by the black hole mass, M_b , by assuming its smallness. M_b is introduced here according to [1],

$$-2\Phi^{(0)}(r=0) = a = \ln\left(\frac{1}{2}QM_b\right). \quad (14)$$

By the reparametrization,

$$-2\Phi^{(0)} = a + \frac{1}{2} \ln [\sinh^2(r)/\beta(r)] = \frac{1}{2}Q\rho. \quad (15)$$

and $\theta' = \sqrt{k'/k}\theta/Q$, the metric $G_{\mu\nu}^{(0)}$ is rewritten for $M_b \ll 1$ and $\rho \geq 0$ as follows,

$$ds^2 = \frac{1}{2}kQ^2\left(\frac{d\rho^2}{\bar{f}(\rho)} + \bar{f}(\rho)d\theta'^2\right), \quad \bar{f}(\rho) = e^{-Q\rho}\sinh^2(r) \approx 4\frac{k}{k'}(1 - \bar{\mu}e^{-\frac{Q}{2}\rho}), \quad (16)$$

where $2\bar{\mu} = QM_b\sqrt{k/k'}$. We can see that (16) is equivalent to an approximate black hole solution found in [15].

This M_b expansion is corresponding to expanding the solutions in terms of the series of $e^{-r_0} \equiv \mu^2$ with the replacement of r by $r + r_0$. The parameter r_0 is defined by (15) at $\rho = 0$ as,

$$a + \frac{1}{2} \ln [\sinh^2(r_0)/\beta(r_0)] = 0. \quad (17)$$

From this and (14), we can see $M_b \propto e^{-r_0}$ for $r_0 \gg 1$. Then M_b expansion is corresponding to examining the equations at large $r(>r_0)$, where the space-time is asymptotically flat but the tail of the black hole can be seen.

From (7) and (8), the linearized equation of $T^{(0)}$ is written as

$$[\partial_r^2 + \frac{1}{2}\partial_r(\ln G^{(0)})\partial_r + \partial_r(-2\Phi^{(0)})\partial_r + k - 2]T^{(0)} = 0, \quad (18)$$

by assuming as $T^{(0)} = T^{(0)}(r)$. This linearization is justified by introducing a small parameter λ as given below. Eq.(18) can be rewritten as,

$$z(1-z)\frac{d^2T^{(0)}}{dz^2} + (c - (a+b+1)z)\frac{dT^{(0)}}{dz} - abT^{(0)} = 0, \quad (19)$$

where $z = \cosh^2(r)$ and

$$a = \frac{1}{4}(1 + \sqrt{9-4k}), \quad b = \frac{1}{4}(1 - \sqrt{9-4k}), \quad c = \frac{1}{2}. \quad (20)$$

The solution of Eq.(19) is known as the hypergeometric function, $F(a, b; c; z) = F(b, a; c; z)$. We firstly solve Eq.(19) for $d < 2$ ($k < 9/4$), then we take the limit of $d = 2$ ($k = 9/4$) of the solution. For large r (or z), it is convenient to consider the following two independent solutions,

$$w_1 = z^{-a}F(a, a-c+1, a-b+1, z^{-1}), \quad w_2 = z^{-b}F(b, b-c+1, b-a+1, z^{-1}). \quad (21)$$

Generally we should take a linear combination of w_1 and w_2 . However, to fix the ratio between them is difficult. Since w_1 and w_2 in $T^{(0)}$ are corresponding to two independent interaction terms on the world surface, the problem of fixing the ratio of them is similar to the "Big Fix" [17] proposed in the 4d quantum gravity in terms of the wormhole interaction. So this might be solved by considering the topology changing interactions of 2d surface. But it is out of our present task. While w_2 is favourable if we demand that $T^{(0)}$ should coincide with the one obtained in [14] in the limit of $r \rightarrow \infty$.⁴ However w_1 can be neglected at large r compared to w_2 even if we retain w_1 through an appropriate linear combination of w_1 and w_2 . So we consider

$$T^{(0)} = w_1 + w_2,$$

as a solution in order to extend this for $d > 2$.

Nextly, we solve the approximate equations of $G_{\mu\nu}$ and Φ in terms of $T^{(0)}$ and the following parametrization [12],

$$\begin{aligned} G_{\mu\nu} &= G_{\mu\nu}^{(0)} + \lambda^2 h_{\mu\nu} + \dots, \quad \Phi = \Phi^{(0)} + \lambda^2 \Phi^{(2)} + \dots, \\ T &= \lambda(T^{(0)} + \lambda T^{(1)} + \dots), \end{aligned} \quad (22)$$

where $h_{\mu\nu} = \text{diag}(0, h(r))$ and λ represents a small parameter. After solving the equations of order λ^2 of (9) and (10) by expanding $G_{\mu\nu}^{(0)}$, $\Phi^{(0)}$ and $T^{(0)}$ in the power

⁴In this limit, the equation for $T^{(0)}$ is written as $[\hbar\partial_r^2 + \partial_r + k - 2]T^{(0)} = 0$, where \hbar is maintained. This equation is solved by assuming the form, $T^{(0)} = e^{-\alpha r}$, and we must choose the solution $\alpha = (1 - \sqrt{1 - 4(k-2)\hbar})/2\hbar$ in order to get from it the classical limit, $\alpha = k - 2$, for $\hbar \rightarrow 0$.

series of μ^2 up to $O(\mu^2)$, we obtain

$$\Phi^{(2)} = \frac{\mu^2}{128} e^{Q_0(1-\epsilon)r}, \quad h = 0. \quad (23)$$

where

$$Q_0 = 2\partial_0 \Phi^{(0)}|_{\mu^2=0} = -\sqrt{\frac{2}{k'}}, \quad \epsilon = \sqrt{9-4k}. \quad (24)$$

Then the 2d solution is obtained by taking the limit of $k = 9/4$ and $\epsilon = 0$. However the results are useful for $d < 2$ (but not for $d > 2$), and the second result, $h = 0$, is independent on ϵ namely on d . This implies that the 2d black hole configuration does not get any modification from the tachyon condensation. While the dilaton is modified. Then a black hole could coexist with the condensed tachyon as an exact string solution. Nextly, we consider the case of $d > 2$ in the following section.

4 $d > 2$; $SL(2, R) \otimes SO(1, 1)^{d-2}/SO(1, 1)$ model

We consider the model $SL(2, R) \otimes SO(1, 1)^{d-2}/SO(1, 1)$ as an example of d -dimensional target space. According to [18], we parametrize the group element of $SL(2, R) \otimes SO(1, 1)^{d-2}$ as,

$$g = \begin{pmatrix} g_0 & 0 & \dots & 0 \\ 0 & g_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & g_{d-2} \end{pmatrix}, \quad g_0 = \begin{pmatrix} p & u \\ -v & q \end{pmatrix}, \quad pq + uv = 1, \quad (25)$$

where g_0 represents the $SL(2, R)$ part, and for the each $SU(1, 1)$,

$$g_i = \begin{pmatrix} \cosh t_i & \sinh t_i \\ \sinh t_i & \cosh t_i \end{pmatrix}, \quad i = 1 \sim d-2. \quad (26)$$

The more convenient parametrizations are given after the gaugings are fixed. The embedding of the subgroup $H = SO(1, 1)$ in G is chosen as,

$$\sigma = \begin{pmatrix} s_0 & 0 & \dots & 0 \\ 0 & s_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & s_{d-2} \end{pmatrix}, \quad s_0 = e_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad s_i = e_i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (27)$$

with the normalization of the coefficients, $\sum_{i=0}^{d-2} e_i^2 = 1$.

In this case, the original action is characterized by $(d-2)$ $SO(1, 1)$ WZW actions $S_W(g_i)$ of level k_i and the previous gauged WZW action of level k . The procedures to obtain the exact metrics are parallel to the previous section.

We firstly consider the case of the vector gauging. Under the vector gauge transformation, $g \rightarrow u^{-1}gu$ and $u \in H$, we can see $\delta p = \delta q = \delta t_i = 0$. This implies that the ground state is independent on u and v . The resulting metric and the dilaton are separately given for two cases, (i) $pq > 0$ and (ii) $pq < 0$. For the case (i), $G_{\mu\nu}^{(0)}$ and $\Phi^{(0)}$ are given as,

$$ds^2 = \frac{k-2}{2} \left(dR^2 - \frac{dX_0^2}{\tanh^2 R - 2/k} + \sum_{i=1}^{d-2} dX_i^2 \right), \quad (28)$$

$$-2\Phi^{(0)} = \text{const} + \ln \sinh 2R - \ln \sqrt{-G^{(0)}}, \quad (29)$$

where $G^{(0)} = \det G_{\mu\nu}^{(0)}$. The $SL(2, R) \otimes SO(1, 1)^{d-2}$ variables are reparametrized as follows;

$$p = \cosh Re^{X_0}, \quad q = \cosh Re^{-X_0}, \quad (30)$$

and X_i ($i = 1 \sim d-2$) are the linear combinations of t_i [18]. In the case of (ii), we obtain the same form with (28) where $\tanh R$ is replaced by $\coth R$ with the parametrization,

$$p = \sinh Re^{X_0}, \quad q = -\sinh Re^{-X_0}. \quad (31)$$

The eq.(29) is common to two cases (i) and (ii) but with different $G^{(0)}$'s.

For the axial vector gauge invariant formalism, the $SL(2, R)$ parameters, u and v , are unchanged under the axial gauge transformation, $g \rightarrow ugu$, then the ground state is expressed in terms of u, v and t_i (namely X_i). We consider the two kinds of metrics in this case also, for (i) $uv > 0$ and for (ii) $uv < 0$. For $uv > 0$,

$$ds^2 = \frac{k-2}{2} \left(dR^2 - \frac{dX_0^2}{(1+\eta)\tanh^2 R - \eta - 2/k} + \frac{dX_{d-2}^2}{1+\eta - 2/k - \eta\coth^2 R} + \sum_{i=1}^{d-3} dX_i^2 \right), \quad (32)$$

in terms of the parametrization (30). Here $\eta = \sum_{i=1}^{d-2} (e_i/e_0)^2 (k_i/k)$. And the metric for $uv < 0$ is given by replacing $\tanh R$ by $\coth R$ in (32) with the parametrization (31), and Φ has the same form with (29) for both the cases with each $G^{(0)}$.

To perform the same analysis done in section 3, we rotate X_0 as $X_0 \rightarrow i\theta$ for the sake of the unitarity. By changing the variable as $2R \equiv r$ and assuming that $T^{(0)}$ depends on r only, we arrive at the same linealized equation of the tachyon with (19) for the above four cases in spite of the difference of $G_{\mu\nu}^{(0)}$ and the dimension. While ϵ becomes pure imaginary here since $(26 >)d > 2$ and

$$9 - 4k = \frac{2-d}{26-d}, \quad (33)$$

which is the condition of the conformal invariance. Then a and b given in (20) are complex conjugate each other. Here we restrict our attention to the real solution since we can not give a physical meaning to the complex solutions. So we choose the same $T^{(0)}$ with that given in section 3,

$$T^{(0)} = w_1 + w_2 \sim 2\sqrt{2}\mu e^{\frac{1}{2}Q_0 r'} \cos\left(\frac{\delta}{2}Q_0 r' + \alpha\right), \quad (34)$$

where $\delta = \sqrt{4k - 9}$ and $r' = r - r_0$. The second expression is the μ expansion stated before.

Next, we solve the equations of other fields in the large $r(> r_0)$ region, where r_0 is defined by (17), by expanding $G_{\mu\nu}^{(0)}$ and $\Phi^{(0)}$ in the series of μ^2 as in the previous section. Here we set the coordinates as $X^\mu = (r, \theta, X^{d-2}, X^i)$, $i = 1 \sim d - 3$, $\mu = 0, 1, 2, \dots, d - 1$, and use eq.(22) with the following parametrization,

$$h_{\mu\nu} = \text{diag}(1, h_1, h_2, \vec{0}), \quad (35)$$

where we take $h_2 = 0$ for vector gauging. After a calculation we obtain the following result which is common to the above four gauged WZW models,

$$\Phi^{(2)} = \left(\frac{\mu^2}{256} \cos(\delta Q_0 r' + 2\alpha) + \phi_c \right) e^{Q_0 r'}, \quad (36)$$

$$(h_1 + h_2) = \frac{4}{Q_0^2} (Q_0^2 \phi_c - \frac{\mu^2}{32}) e^{Q_0 r'}, \quad (37)$$

where ϕ_c is a constant which can not be determined by solving the equations.

In order to decide ϕ_c , we require a reasonable condition that $\Phi^{(2)}$ should coincide with that of 2d ((23)) when we take the limit $d \rightarrow 2$ in eq.(36). From this requirement, we get

$$\phi_c = \frac{\mu^2}{256}. \quad (38)$$

Except for the undetermined ratio of h_1 and h_2 , we finally obtain the following results,

$$\Phi^{(2)} = \frac{\mu^2}{256} [1 + \cos(\delta Q_0 r' + 2\alpha)] e^{Q_0 r'}, \quad (h_1 + h_2) = -\frac{\mu^2}{64} \frac{d-2}{26-d} e^{Q_0 r'}. \quad (39)$$

We notice that $h_1 + h_2$ vanishes at $d = 2$ limit, and this is consistent with the 2d result (23).

In order to continue (39) to $d \leq 2$, we should consider the case of vector gauging where we could take as $h_1 \equiv h$ and $h_2 = 0$. The previous result (23) shows that $h = 0$ for $d \leq 2$, but (39) does not lead to this result. So we can not apply (39) for $d < 2$. The same thing is said for $\Phi^{(0)}$ also. This implies that there are different

phases above and below the critical dimension $d = 2$. We should also notice the next curious point. For $d > 2$, both corrections h and $\Phi^{(2)}$ appear, but the behavior of damping oscillation of $T^{(0)}$ with r is reflected on $\Phi^{(2)}$ only and not on h .

The result of non-zero $h_1 + h_2$ implies that the tachyon condensation disturbs the singular metric configurations for $d > 2$. The metric $G_{\mu\nu}^{(0)}$'s are expanded by μ^2 as follows,

$$G_{\mu\nu}^{(0)} = \text{diag}(1, 1 \pm 4\mu^2 e^{Q_0 r'}, 0, \vec{0}) + O(\mu^4), \quad (40)$$

for the vector gauge, and

$$G_{\mu\nu}^{(0)} = \text{diag}(1, 1 \pm 4\mu^2(1 + \eta)e^{Q_0 r'}, 1 \pm 4\mu^2\eta e^{Q_0 r'}, \vec{0}) + O(\mu^4), \quad (41)$$

for the axial vector gauge. The upper (lower) signs are obtained for the parametrization (30) ((31)). Then we can say that the tail of the original, singular metric, $G_{\mu\nu}^{(0)}$, of $O(\mu^2)$ could be canceled out by h_i by choosing an appropriate value of λ . Even if we could not cancel out the higher order terms of μ^2 in $G_{\mu\nu}^{(0)}$ by the higher order corrections due to tachyon condensation, the final form of the modified $G_{\mu\nu}$ could not maintain the original structure of $G_{\mu\nu}^{(0)}$, i.e. the black hole (string) structure. In this sense, the singularity of space-time seems to be eliminated through the tachyon condensation for $d > 2$, especially at the realistic dimension $d = 4$. This point may be an important result. However we need more higher order calculations to assure this point. We will discuss on this point elsewhere.

5 Conclusion

Here we find that the 2d black hole is not affected by the condensation of the tachyon, at least up to the quadratic order of the tachyon field. On the other hand, the dilaton gets a modification. However for $d > 2$, both the metric and the dilaton are modified, and these modifications are coinciding with that of 2d case at the limit of $d = 2$. However the solutions obtained for $d > 2$ can not be continued to the region of $d < 2$ and vice versa. Then we might say that a kind of phase transition, where the order parameter is the shift of the metric from the WZW solution, has happened at the critical dimension $d = 2$. This implies that the singularities of the space-time manifold could be removed by considering the tachyon condensed vacuum for $d > 2$.

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